AA214 – Fall 1996– Problem Set 2

A second-order backward (or "upwind") differencing approximation to a 1st derivative is given as a point operator by

$$\left(\frac{\partial u}{\partial x}\right)_j = \frac{1}{2\Delta x}(u_{j-2} - 4u_{j-1} + 3u_j) \tag{1}$$

and can be analyzed as in Chapter 3 in the class notes.

1. Express Eq. 1 in banded matrix form, then derive the symmetric and skew symmetric matrices that have it as their sum. (See Appendix A for constructings symmetric and skew symmetric matrices)

2. Construct a Taylor table for both the symmetric and skew symmetric matrices in Prob 1, and find er_t for both. (In this case, find the derivative which is approximated by the symmetric and skew-symmetric operators.)

3. Write all the elements in a 7x7 matrix operator that expresses the skew symmetric form with <u>periodic</u> (bc).

4. Find, by means of a Taylor table, the values of a, b, c, and d that minimize the value of er_t in the expression

$$a\left(\frac{\partial u}{\partial x}\right)_{j-1} + \left(\frac{\partial u}{\partial x}\right)_j - \frac{1}{\Delta x}[bu_{j+1} + cu_j + du_{j-1}] = ? \tag{2}$$

   What is the resulting finite difference scheme and what is the value of er_t?

5. Using a 4 (interior) point mesh, write out the 4×4 matrices and the (bc) vector formed by using the above scheme when both u and $\partial u/\partial x$ are given at $j = 0$ and u is given at $j = 5$.

6. Repeat 4 when $b = 0$. This is an example of an upwind Padé scheme.